ST109 Class 10 of Week 5

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Total Probability Formula

- Expresses the total probability of an outcome which can be realized via several distinct events, hence the name.
- Suppose $B_1, B_2, \ldots, B_k$ form a partition of the sample space. Therefore, $A \cap B_1, A \cap B_2, \ldots, A \cap B_K$, form a partition of A.

$$\begin{aligned}\mathbb{P}(A) &= \sum_{i=1}^{K} \mathbb{P}(A \cap B_i) \\ &= \sum_{i=1}^{K} \mathbb{P}(A \mid B_i)\mathbb{P}(B_i)\end{aligned}$$

- These distinct events could be treated as **causes** of A, and the total probability could be recognized as the **weighted average** in terms of the probabilities of those causes.

Bayes' Theorem

$$\mathbb{P}(B_j \mid A) = \frac{\mathbb{P}(B_j \cap A)}{\mathbb{P}(A)}$$
$$= \frac{\mathbb{P}(A \mid B_j)\mathbb{P}(B_j)}{\sum_{i=1}^{K} \mathbb{P}(A \mid B_i)\mathbb{P}(B_i)}$$

which holds for each $B_j,\ j = 1, 2, \ldots, K$.

- A has occurred.
- The intuition behind the Bayes' Theorem is to find the probability of the underlying **causes** of A, i.e., $\mathbb{P}(B_j \mid A),\ j = 1, 2, \ldots, K$.

Probability Function

- **Random variable** X is an experiment for which the outcomes are numbers, i.e., a function from the sample space S mapping to the real number $\mathbb{R}$.
- The **probability function (pf)** of a discrete random variable X, denoted by $p(x)$, is a real-valued function such that for any number x the function is:

$$p(x) = \mathbb{P}(X = x).$$

1. $p(x)0$ for all real numbers x.
2. $\sum_{x_i \in S} p(x_i) = 1$, i.e. the sum of probabilities of all possible values of X is 1.