



ST109 Class 10 of Week 7

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PDF & CDF

PDF satisfies the following properties:

- ▶ $f_X(x) \geq 0$ for all x .
- ▶ $\int_x f_X(x)dx = 1$.

CDF satisfies the following properties:

- ▶ $F_X(x) \in [0, 1]$.
- ▶ $\lim_{x \rightarrow -\infty} F_X(x) = 0, \lim_{x \rightarrow \infty} F_X(x) = 1$.
- ▶ $F_X(x)$ is non-decreasing.
- ▶ $F_X(x)$ is right-continuous.

The relationship between PDF & CDF:

- ▶ $\int_{-\infty}^x f_X(x)dx = F_X(x)$.
- ▶ $F'_X(x) = f_X(x)$.

Moment

n-th moment:

$$\mathbb{E}(X^n) = \int_{-\infty}^{\infty} x^n f_X(x) dx$$

MGF:

$$M_X(t) = \mathbb{E}(e^{tX}) = \begin{cases} \sum_x e^{tx} f_X(x), & X \text{ is discrete} \\ \int_x e^{tx} f_X(x) dx, & X \text{ is continuous} \end{cases}$$

MGF $\rightarrow$ n-th moment:

$$\mathbb{E}(X^n) = M_X^{(n)}(0) = \left. \frac{d^n M_X(t)}{dt^n} \right|_{t=0}$$

(Try to prove it. Hint: Consider the Taylor expansion of e^{tX})