ST109 Class 10 of Week 8

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Discrete Uniform Distribution

Definition

X has a discrete uniform distribution if all of these values have the same probability, i.e.:

$$\mathbb{P}(X = x) = \begin{cases} 1/k, & x = 1, \ldots, k \\ 0, & \text{otherwise} \end{cases}$$

Mean & Variance

$$\mathbb{E}(X) = \sum_{x=1}^{k} x\, p(x) = \frac{1 + 2 + \cdots + k}{k} = \frac{k+1}{2}$$

$$\mathbb{E}(X^2) = \sum_{x=1}^{k} x^2 p(x) = \frac{1^2 + 2^2 + \cdots + k^2}{k} = \frac{(k+1)(2k+1)}{6}$$

$$\mathrm{var}(X) = \mathbb{E}(X^2) - [(\mathbb{E}(X))^2] = \frac{k^2 - 1}{12}$$

Bernoulli Distribution

Definition

$X \sim \text{Bernoulli}(p)$ if X takes the value 1 with probability p and the value 0 with probability $q = 1 - p$, i.e.:

$$\mathbb{P}(X = x) = \begin{cases} p^x(1-p)^{1-x}, & x \in \{0, 1\} \\ 0, & \text{otherwise} \end{cases}$$

Mean & Variance

$$\mathbb{E}(X) = p$$
$$\text{var}(X) = p(1-p)$$

Moment Generating Function

$$M_X(t) = \sum_{x=0}^{1} e^{tx} p(x) = (1-p) + pe^t$$

Binomial Distribution

Definition

$X \sim \text{Binomial}(n, p)$ if X is the number of successes in a sequence of n independent Bernoulli trial with success probability p, i.e.:

$$\mathbb{P}(X = x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

Mean & Variance

$$\mathbb{E}(X) = np$$
$$\text{var}(X) = np(1-p)$$

Moment Generating Function

$$M_X(t) = \sum_{x=0}^{1} e^{tx} p(x) = ((1-p) + pe^t)^n$$

Poisson Distribution

Definition

$X \sim \text{Poisson}(\lambda)$ if X expresses the probability of a given number of events occurring in a fixed interval of time if these events occur with a known constant mean rate λ and independently of the time since the last event, i.e.:

$$\mathbb{P}(X = x) = \begin{cases} e^{-\lambda}\lambda^x / x!, & x = 0, 1, \dots \\ 0, & \text{otherwise} \end{cases}$$

Mean & Variance

$$\mathbb{E}(X) = \text{var}(X) = \lambda$$

Moment Generating Function

$$M_X(t) = e^{\lambda(e^t - 1)}$$