ST109 Class 10 of Week 9

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Nov 28, 2024

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Uniform Distribution

Definition

$X \sim \text{Unif}(a, b)$ if the outcome of X is arbitrary that lies in $[a, b]$, i.e.:

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & x \in [a, b] \\ 0, & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & x \in [a, b] \\ 1, & x > b \end{cases}$$

Mean & Variance

$$\mathbb{E}(X) = \int_a^b x \frac{1}{b-a} \mathrm{d}x = \frac{a+b}{2}$$

$$\text{var}(X) = \frac{(b-a)^2}{12}$$

Moment Generating Function

$$M_X(t) = \mathbb{E}(e^{tX}) = \frac{e^{tb} - e^{ta}}{t(b-a)}$$

Exponential Distribution

Definition

$X \sim \text{Exp}(\lambda)$ if the distance between events is in a Poisson point process, i.e.:

$$f_X(x) = \begin{cases} \lambda e^{\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\lambda x}, & x \geq 0 \end{cases}$$

Mean & Variance

$$\mathbb{E}(X) = \int_0^\infty x\lambda e^{\lambda x} \mathrm{d}x = \frac{1}{\lambda}$$

$$\text{var}(X) = \frac{1}{\lambda^2}$$

Moment Generating Function

$$M_X(t) = \mathbb{E}(e^{tX}) = \frac{\lambda}{\lambda - t}$$

Normal Distribution

Definition

$X \sim N(\mu, \sigma^2)$ if:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ \frac{(x-\mu)^2}{2\sigma^2} \right\}$$

Mean & Variance

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ \frac{(x-\mu)^2}{2\sigma^2} \right\} \mathrm{d}x = \mu$$

$$\text{var}(X) = \sigma^2$$