ST326 Week 2

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About me

Education:

- Second year PhD student.
- Supervision under Prof. Clifford Lam and Dr. Yunxiao Chen.
- Research interests in: High-dimensional time series analysis; Latent factor model with vector, matrix and tensor time series.
- Obtained BEcon in major of Economic Statistics from Beihang University (formerly known as Beijing University of Aeronautics and Astronautics) in 2023, and MSc in major of Statistics (Financial Statistics) from LSE in 2024.

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Strong/Strict stationarity

Definition (Strong/Strict stationarity)

For any arbitrary time interval $t_1, \ldots, t_m$ with length m, for any time lag $h \in \mathbb{Z}$, we say $\{x_t\}_{t=1}^{T}$ is **strongly/strictly stationary** if the joint distributions of $(x_{t_1}, \ldots, x_{t_m})$ and $(x_{t_1+h} \ldots, x_{t_m+h})$ coincide, i.e.,

$$\mathbb{P}(x_{t_1} \leq a_1, \cdots, x_{t_m} \leq a_m) = \mathbb{P}(x_{t_1+h} \leq a_1, \cdots, x_{t_m+h} \leq a_m)$$

for any constants $a_1, \ldots, a_m$.

Some intuitions (not limited to):

- $m = 1$: $\mathbb{E}(x_t) = \mathbb{E}(x_{t+h})$ if 1-st order moments exist.
- $m = 2$: $\mathrm{Cov}(x_{t_1}, x_{t_2}) = \mathrm{Cov}(x_{t_1+h}, x_{t_2+h})$ if 2-nd order moments exist.
- $\ldots$

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Weak stationarity

Definition (Weak stationarity)

We say $\{x_t\}$ is weakly stationary (or just "stationary") if, for $t \in \{1, \ldots, T\}$

- $\mu = \mathbb{E}(x_t) < \infty$, and
- For any $h \in \mathbb{Z}$, $\gamma(h) = \mathrm{Cov}(x_t, x_{t+h}) < \infty$.

An example: If $x_t \sim \text{i.i.d.}(\mu, \sigma^2)$, then $\{x_t\}$ is weakly stationary.

Bridging weak and strong stationarity

Definition (Gaussian Process (GP))

We say $\{x_t\}$ is a Gaussian Process if for any $m > 0$, for any time points $t_1, \ldots, t_m$, the vector $\mathbf{x} = (x_{t_1}, \ldots, x_{t_m})$ has a multivariate normal distribution $N(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$ with finite means and variances for components.

FYI: $\boldsymbol{\mu}_t = \mathbb{E}(\mathbf{x}_t)$ and $\boldsymbol{\Sigma}_t = \big(\mathrm{Cov}(x_{t_i}, x_{t_j})\big)$

Theorem (GP + weak stationarity $\implies$ strong stationarity)

If $\{x_t\}$ is both a Gaussian process and weakly stationary, then $\{x_t\}$ is strongly stationary.

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Important tools for time series analysis

Autocovariance Sequence (ACVS)

Define ACVS for stationary time series $\{x_t\}$ as s_h, where

$$s_h = \mathrm{Cov}(x_t, x_{t+h}) = \mathrm{Cov}(x_{t+h}, x_t)$$

Autocorrelation Function (ACF)

Define ACF for stationary time series $\{x_t\}$ as $\gamma(h)$, where

$$\gamma(h) = \frac{s_h}{s_0} = \frac{\mathrm{Cov}(x_t, x_{t+h})}{\sqrt{\mathrm{Var}(x_t)\mathrm{Var}(x_{t+h})}}$$