ST418 Week 3

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White noise process

Definition (White noise process, WN)

If $\{x_t\}$ is a white noise process, then

- $\mathbb{E}(x_t) = \mu < \infty$,
- $s_\tau = 0$ for $\tau \neq 0$, and $s_0 = \sigma^2 < \infty$ (time-independent).

Shorthand: $x_t \sim \text{WN}(\mu, \sigma^2)$.

Examples:

- $x_t \overset{\text{i.i.d.}}{\sim} N(0, 1)$ (Gaussian white noise).
- Uncorrelated but not i.i.d.: e.g. alternating distributions with the *same mean* and *finite variance*.

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Moving average process: MA(q)

Definition (Moving average process of order q, MA(q))

$\{x_t\}$ is MA(q) if

$$x_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q},$$

where $\theta_q \neq 0$ and $\varepsilon_t \sim \mathrm{WN}(0, \sigma_\varepsilon^2)$.

Key facts:

- Any finite-order MA(q) is **weakly stationary**.
- **Cutoff**: $s_\tau = 0$ and $\rho(\tau) = 0$ for $|\tau| > q$.

Autoregressive process: AR(p)

Definition (Autoregressive process of order p, AR(p))

$\{x_t\}$ is AR(p) if

$$x_t = \alpha + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + \varepsilon_t,$$

where $\phi_p \neq 0$ and $\varepsilon_t \sim \text{WN}(0, \sigma_\varepsilon^2)$.

Key facts (for review):

- AR(p) is **not automatically** stationary (stationarity conditions come later).
- When stationary: ACF typically **tails off** (often exponential-like decay).

PACF: definition via “removing intermediates”

Definition (Partial autocorrelation function, PACF)

For a mean-0 stationary process $\{x_t\}$, define

$$\begin{aligned}
\pi(1) &= \mathrm{Corr}(x_2, x_1), \\
\pi(2) &= \mathrm{Corr}\big(x_3 - \widehat{\mathbb{E}}(x_3|x_2),\ x_1 - \widehat{\mathbb{E}}(x_1|x_2)\big), \\
\pi(3) &= \mathrm{Corr}\big(x_4 - \widehat{\mathbb{E}}(x_4|x_3, x_2),\ x_1 - \widehat{\mathbb{E}}(x_1|x_3, x_2)\big),\ \ldots
\end{aligned}$$

- $\widehat{\mathbb{E}}(\cdot|\cdot)$ is the **best linear predictor** (least squares projection).
- Interpretation: $\pi(k)$ is the correlation between the **residual parts** of x_t and x_{t-k} after removing linear effects of the intermediate lags.
- In practice: estimate via linear regression residuals.

Identify AR vs MA using ACF/PACF

Key fingerprints

AR(p)	**MA(q)**
$\pi(k) = 0$ for $k > p$	$\rho(k) = 0$ for $k > q$

- **AR(p):** PACF *cuts off* at lag p; ACF *tails off*.
- **MA(q):** ACF *cuts off* at lag q; PACF *tails off*.

Practical takeaway: use sample ACF/PACF plots to propose candidate orders (p, q), then fit/diagnose.