



ST326 Week 2

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Stylised Facts (I)

Near stationarity and heavy tails

- ▶ **Near stationarity:** returns fluctuate around a stable mean and are often modelled as weakly stationary: $\mathbb{E}(r_t) = \mu$ and $\text{Cov}(r_t, r_{t-k}) = \gamma(k)$.
- ▶ **Heavy tails:** extreme returns occur more often than a Normal distribution predicts.

Asymmetry and volatility clustering

- ▶ **Asymmetry:** return distributions are often negatively skewed, with more pronounced downside extremes.
- ▶ **Volatility clustering:** large absolute returns tend to follow large absolute returns, and small ones tend to follow small ones.

Stylised Facts (II)

Aggregational Gaussianity

As the holding period increases, aggregated returns tend to become closer to Normal: $r_t(k) = \sum_{j=0}^{k-1} r_{t-j}$.

Long-range dependence

Raw returns have little autocorrelation, whereas squared and absolute returns can display persistent autocorrelation: $\text{Corr}(r_t, r_{t-k}) \approx 0$, but $\text{Corr}(r_t^2, r_{t-k}^2)$ can be nonzero.

Leverage effect

Negative returns tend to be associated with an increase in subsequent volatility.

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Efficient Markets Hypothesis (EMH)

Economic idea

Asset prices rapidly incorporate available information. Under weak-form EMH, historical prices should not create systematic opportunities for abnormal risk-adjusted returns.

Simplified statistical model used in this course

$$r_t = \mu + \varepsilon_t, \quad \mathbb{E}(\varepsilon_t \mid \mathcal{F}_{t-1}) = 0.$$

Returns are unpredictable in conditional mean, while volatility may still be predictable. EMH does not require returns to be Normal or IID.

IID, Martingale Difference and White Noise

Definitions

- ▶ **IID:** independent across time and identically distributed, with zero mean and finite, constant variance.
- ▶ **Martingale difference (MD):** $\mathbb{E}(\varepsilon_t \mid \mathcal{F}_{t-1}) = 0$; no predictability in conditional mean from past information.
- ▶ **White noise (WN):** $\mathbb{E}(\varepsilon_t) = 0$, $\text{Var}(\varepsilon_t) = \sigma^2$, and $\text{Cov}(\varepsilon_t, \varepsilon_{t-k}) = 0$ for $k \neq 0$.

Logical relationship

$$\boxed{\text{IID} \implies \text{MD} \implies \text{WN}}$$

The converses do not generally hold.

EMH motivates WN; data are more consistent with MD than IID: mean returns can be unpredictable when volatility is predictable.

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Autocorrelation Function (ACF)

For a weakly stationary time series $\{X_t\}$, the ACF measures linear dependence at lag k .

Population ACF

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} = \frac{\text{Cov}(X_t, X_{t-k})}{\text{Var}(X_t)}.$$

Sample ACF

$$\hat{\rho}(k) = \frac{\hat{\gamma}(k)}{\hat{\gamma}(0)},$$
$$\hat{\gamma}(k) = \frac{1}{T} \sum_{t=k+1}^T (X_t - \bar{X})(X_{t-k} - \bar{X}), \quad \bar{X} = \frac{1}{T} \sum_{t=1}^T X_t.$$

Ljung–Box Portmanteau Test

Hypotheses

$$H_0 : \rho(1) = \rho(2) = \cdots = \rho(m) = 0,$$

$$H_1 : \rho(k) \neq 0 \quad \text{for at least one } k \in \{1, \dots, m\}.$$

Test statistic

$$Q_m = T(T+2) \sum_{j=1}^m \frac{\hat{\rho}(j)^2}{T-j}.$$

This aggregates squared sample autocorrelations over the first m lags.

Under suitable null assumptions, $Q_m \overset{a}{\sim} \chi_m^2$. Reject H_0 at level α when $Q_m > \chi_{\alpha, m}^2$, the upper- α critical value.

Dickey–Fuller Test

General specification

$$X_t = \mu + \beta t + \alpha X_{t-1} + \varepsilon_t, \quad X_t = \log P_t.$$

Hypotheses

$$H_0 : \alpha = 1 \quad (\text{unit root}), \quad H_1 : \alpha < 1.$$

The stationary alternative is subject to the deterministic specification.

Test statistic

$$W = \frac{\hat{\alpha} - 1}{SE(\hat{\alpha})}.$$

Estimate by OLS; use Dickey–Fuller critical values rather than ordinary t critical values; reject for sufficiently negative values.

Jarque–Bera Test for Normality

Sample skewness and kurtosis

$$S = \frac{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^3}{\hat{\sigma}^3}, \quad K = \frac{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^4}{\hat{\sigma}^4},$$
$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2.$$

Hypotheses and statistic

H_0 : the observations are IID Normal, so $S = 0$ and $K = 3$. H_1 : departure from Normality through skewness and/or kurtosis.

$$JB = \frac{n}{6} \left[S^2 + \frac{(K - 3)^2}{4} \right].$$

Under the IID Normal null, $JB \overset{a}{\sim} \chi^2_2$. Reject H_0 at level α when $JB > \chi^2_{\alpha,2}$, the upper- α critical value.